AN5019

Magnetic Calibration Algorithms

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Document information

Info	Content
Abstract	This application note documents mathematical approximations to inverse trigonometric functions used in the NXP Sensor Fusion Library and contained in the file *magnetic.c* .

Revision history

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AN5019 v2.0	20160621	AN5019 v1.0
Modifications:	• Minor changes • The format of this document has been redesigned to comply with the new identity guidelines of NXP Semiconductors. Legal texts have been adapted to the new company name where appropriate.	
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Contact information

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1. Introduction

1.1 Summary

This application note documents the mathematics for magnetic calibration used by the NXP Sensor Fusion Library. These functions compute the calibration of the magnetometer sensor with respect to the hard and soft iron magnetic interference caused by ferromagnetic components on the circuit board.

Section 2 derives the mathematical techniques used throughout in this document for least squares optimization. Section 3 defines the general hard and soft iron magnetic interference model. Sections 4 to 6 derive the solutions for three models of magnetic calibration in order of increasing complexity.

1.2 Terminology

Table 1. Terminology

Term/Symbol	Definition
Left superscript G	Denotes that the measurement is in the global frame: eg ${}^{G}\boldsymbol{B}_k$
Left superscript S	Denotes that the measurement is in the sensor frame: eg ${}^{S}\boldsymbol{B}_k$
$\boldsymbol{A}$	Ellipsoid matrix: $\boldsymbol{A} = (\boldsymbol{W}^{-1})^T\boldsymbol{W}^{-1}$
B	Magnitude of the geomagnetic vector ${}^{G}\boldsymbol{B}_0$
${}^{G}\boldsymbol{B}_0$	Geomagnetic vector measured in the global frame
${}^{S}\boldsymbol{B}_k$	Uncalibrated magnetometer measurement k in the sensor frame
${}^{S}\boldsymbol{B}_{c,k}$	Calibrated magnetometer measurement k in the sensor frame
E	Error function
M	Number of measurements used in calibration fit
N	Number of calibration model coefficients
$\boldsymbol{Q}$	Matrix of eigenvectors
r_k	Error residual in measurement k
$\boldsymbol{r}$	Vector of residual errors
$\boldsymbol{R}$	Rotation matrix
$\boldsymbol{V}$	Hard iron offset vector
$\boldsymbol{W}$	Soft iron gain matrix
$X_{j,k}$	j-*th* dependent variable in fit to magnetometer measurement k
$\boldsymbol{X}$	Matrix of magnetometer measurements

Term/Symbol	Definition
Y_k	Dependent variable in magnetometer measurement k
$\boldsymbol{\beta}$	Solution vector
λ	Eigenvalue and Lagrange Multiplier
$\boldsymbol{\Lambda}$	Diagonal matrix of eigenvalues

1.3 Software Functions

Table 2. Software Functions

Function	Description	Section
`void fInvertMagCal(struct MagSensor *pthisMag, struct MagCalibration *pthisMagCal)`	Computes calibrated magnetometer measurements by applying the calibration coefficients to uncalibrated measurements.	3.1
`void fUpdateMagCalibration4Slice(struct MagCalibration *pthisMagCal, struct MagBuffer *pthisMagBuffer, struct MagSensor *pthisMag)`	Determines the coefficients of the 4 parameter calibration model using a time slice algorithm.	4, 7
`void fUpdateMagCalibration7Slice(struct MagCalibration *pthisMagCal, struct MagBuffer *pthisMagBuffer, struct MagSensor *pthisMag)`	Determines the coefficients of the 7 parameter calibration model using a time slice algorithm.	5, 7
`void fUpdateMagCalibration10Slice(struct MagCalibration *pthisMagCal, struct MagBuffer *pthisMagBuffer, struct MagSensor *pthisMag)`	Determines the coefficients of the 10 parameter calibration model using a time slice algorithm.	6, 7

2. Least Squares Optimization

2.1 Linear Measurement Model

The general linear model relating the dependent variable Y_k to the independent variables $X_{j,k}$ at measurement k through N model parameters β_j is:

$$Y_k = \beta_0 X_{0,k} + \beta_1 X_{1,k} + \cdots + \beta_{N-1} X_{N-1,k} \tag{1}$$

The fit to the model will not be perfectly accurate and will result in an error residual r_k defined as:

$$r_k = Y_k - \beta_0 X_{0,k} - \beta_1 X_{1,k} - \cdots - \beta_{N-1} X_{N-1,k} \tag{2}$$

For a series of M measurements, equation (2) can be written in matrix form as:

$$\begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{M-1} \end{pmatrix} = \begin{pmatrix} Y_0 \\ Y_1 \\ \dots \\ Y_{M-1} \end{pmatrix} - \begin{pmatrix} X_{0,0} & X_{1,0} & \dots & X_{N-1,0} \\ X_{0,1} & X_{1,1} & \dots & X_{N-1,1} \\ \dots & \dots & \dots & \dots \\ X_{0,M-1} & X_{1,M-1} & \dots & X_{N-1,M-1} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \dots \\ \beta_{N-1} \end{pmatrix} \tag{3}$$

With the definitions that $\boldsymbol{r}$ is the $M \times 1$ column vector of error residuals:

$$\boldsymbol{r} = \begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{M-1} \end{pmatrix} \tag{4}$$

and $\boldsymbol{Y}$ is the $M \times 1$ column vector of M measurements of the dependent variable:

$$\boldsymbol{Y} = \begin{pmatrix} Y_0 \\ Y_1 \\ \dots \\ Y_{M-1} \end{pmatrix} \tag{5}$$

and $\boldsymbol{X}$ is the $M \times N$ matrix containing the M measurements of the independent variable:

$$\boldsymbol{X} = \begin{pmatrix} X_{0,0} & X_{1,0} & \dots & X_{N-1,0} \\ X_{0,1} & X_{1,1} & \dots & X_{N-1,1} \\ \dots & \dots & \dots & \dots \\ X_{0,M-1} & X_{1,M-1} & \dots & X_{N-1,M-1} \end{pmatrix} \tag{6}$$

and $\boldsymbol{\beta}$ is the $N \times 1$ column vector of unknown model coefficients β_0 to β_{N-1} to be determined:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \dots \\ \beta_{N-1} \end{pmatrix} \tag{7}$$

then equation (3) can be written in the form:

$$\boldsymbol{r} = \boldsymbol{Y} - \boldsymbol{X\beta} \tag{8}$$

If there are more measurements M than there are unknowns N, then the equations are solved in a least squares sense by minimizing the error function $E(\boldsymbol{\beta})$ defined as the modulus squared of the error vector $\boldsymbol{r}$ defined in equation (5):

$$E(\boldsymbol{\beta}) = \sum_{k=0}^{M-1} {r_k}^2 = \|\boldsymbol{r}\|^2 = \boldsymbol{r}^T\boldsymbol{r} = (\boldsymbol{Y} - \boldsymbol{X\beta})^T(\boldsymbol{Y} - \boldsymbol{X\beta}) = \|\boldsymbol{Y} - \boldsymbol{X\beta}\|^2 \quad \textbf{(9)}$$

2.2 Normal Equations Solution in Non-Homogeneous Case

If the measurement vector $\boldsymbol{Y}$ is non-zero, then the equations are termed non-homogeneous. The error function $E(\boldsymbol{\beta})$ will be a minimum when it is stationary with respect to any perturbation $\delta\boldsymbol{\beta}$ about the optimal least squares solution $\boldsymbol{\beta}$:

$$lim_{\delta\boldsymbol{\beta}\to\boldsymbol{0}}\{E(\boldsymbol{\beta} + \delta\boldsymbol{\beta}) - E(\boldsymbol{\beta})\} = 0 \; for \; all \; \delta\boldsymbol{\beta} \quad \textbf{(10)}$$

Equation (9) shows that the error function is a simple quadratic function of the model coefficients $\boldsymbol{\beta}$ leading to a single global minimum where the error function is stationary with respect to perturbations. There are no local minima in the error function.

Substituting equation (8) into (10) and ignoring second order terms gives:

$$\{\boldsymbol{Y} - \boldsymbol{X}(\boldsymbol{\beta} + \delta\boldsymbol{\beta})\}^T\{\boldsymbol{Y} - \boldsymbol{X}(\boldsymbol{\beta} + \delta\boldsymbol{\beta})\} - (\boldsymbol{Y} - \boldsymbol{X\beta})^T(\boldsymbol{Y} - \boldsymbol{X\beta}) = 0 \quad \textbf{(11)}$$

$$\Rightarrow -\boldsymbol{Y}^T\boldsymbol{X}\delta\boldsymbol{\beta} + (\boldsymbol{X\beta})^T\boldsymbol{X}\delta\boldsymbol{\beta} - (\boldsymbol{X}\delta\boldsymbol{\beta})^T\boldsymbol{Y} + (\boldsymbol{X}\delta\boldsymbol{\beta})^T\boldsymbol{X\beta} = 0 \quad \textbf{(12)}$$

$$\Rightarrow -\boldsymbol{Y}^T\boldsymbol{X}\delta\boldsymbol{\beta} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X}\delta\boldsymbol{\beta} - \delta\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{Y} + \delta\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} = 0 \quad \textbf{(13)}$$

Each component of equation (13) is a scalar and is therefore unchanged by the transpose operation allowing equation (13) to be rewritten as:

$$2(-\boldsymbol{Y}^T\boldsymbol{X} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X})\delta\boldsymbol{\beta} = 0 \; for \; all \; \delta\boldsymbol{\beta} \quad \textbf{(14)}$$

$$\Rightarrow \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X} = \boldsymbol{Y}^T\boldsymbol{X} \Rightarrow \boldsymbol{X}^T\boldsymbol{X\beta} = \boldsymbol{X}^T\boldsymbol{Y} \quad \textbf{(15)}$$

$$\Rightarrow \boldsymbol{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y} \quad \textbf{(16)}$$

Equation (16) is termed the Normal Equations solution for $\boldsymbol{\beta}$ in the non-homogeneous case. Expanding the expression for the error function $E(\boldsymbol{\beta})$ defined in equation (9) gives:

$$E(\boldsymbol{\beta}) = (\boldsymbol{Y}^T - \boldsymbol{\beta}^T\boldsymbol{X}^T)(\boldsymbol{Y} - \boldsymbol{X\beta}) = \boldsymbol{Y}^T\boldsymbol{Y} - \boldsymbol{Y}^T\boldsymbol{X\beta} - \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{Y} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} \quad \textbf{(17)}$$

Each term of equation (17) is again a scalar and equal to its transpose. The error function $E(\boldsymbol{\beta})$ can therefore be written as:

$$E(\boldsymbol{\beta}) = \boldsymbol{Y}^T\boldsymbol{Y} - 2\boldsymbol{Y}^T\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{\beta} = \boldsymbol{Y}^T\boldsymbol{Y} - 2\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{Y} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{\beta} \quad (18)$$

For the special case where the number of measurements M equals the number of model parameters N to be fitted, the matrix $\boldsymbol{X}$ is square and the transpose and inversion operators commute:

$$(\boldsymbol{X}^T\boldsymbol{X})^{-1} = \boldsymbol{X}^{-1}(\boldsymbol{X}^T)^{-1} \quad (19)$$

Equation (16) can then be written as:

$$\boldsymbol{\beta} = \boldsymbol{X}^{-1}(\boldsymbol{X}^T)^{-1}\boldsymbol{X}^T\boldsymbol{Y} = \boldsymbol{X}^{-1}\boldsymbol{Y} \quad (20)$$

and the error function $E(\boldsymbol{\beta})$ evaluates to zero in this case:

$$E(\boldsymbol{\beta}) = \boldsymbol{Y}^T\boldsymbol{Y} - 2\boldsymbol{Y}^T\boldsymbol{X}\boldsymbol{X}^{-1}\boldsymbol{Y} + (\boldsymbol{X}^{-1}\boldsymbol{Y})^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{X}^{-1}\boldsymbol{Y} = \boldsymbol{Y}^T\boldsymbol{Y} - 2\boldsymbol{Y}^T\boldsymbol{Y} + \boldsymbol{Y}^T(\boldsymbol{X}^T)^{-1}\boldsymbol{X}^T\boldsymbol{Y} = 0 \quad (21)$$

Equation (21) states the expected result that the error function is zero and the fit is perfect when the number of model parameters to be fitted N equals the number of measurements M.

2.3 Constrained Optimization Via Lagrange Multipliers

The method of Lagrange Multipliers is a standard mathematical technique for constrained optimization of the scalar field $f(\boldsymbol{x})$ defined in the N dimensional space $\boldsymbol{x}$ subject to the constraint $g(\boldsymbol{x}) = c$ where c is constant.

The *unconstrained* stationary points of f occur when $\nabla_{\boldsymbol{x}} f(\boldsymbol{x}) = 0$ but this solution does not, in general, satisfy the constraint $g(\boldsymbol{x}) = c$.

The method of Lagrange Multipliers states that the *constrained* stationary point occurs where the gradient vectors of $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are parallel:

$$\nabla_{\boldsymbol{x}} f(\boldsymbol{x}) = -\lambda \nabla_{\boldsymbol{x}} g(\boldsymbol{x}) \quad (22)$$

where the unknown parameter λ is termed the Lagrange Multiplier. The proof follows.

The gradient vector $\nabla_{\boldsymbol{x}} g(\boldsymbol{x})$ of the scalar field $g(\boldsymbol{x})$ is normal to any displacement $\Delta\boldsymbol{x}$ lying on the constraint surface $g(\boldsymbol{x}) = c$:

$$\nabla_{\boldsymbol{x}} g(\boldsymbol{x}).\Delta\boldsymbol{x} = 0 \quad (23)$$

Since, by definition, the solution requires that $f(\boldsymbol{x})$ be stationary subject to the constraint $g(\boldsymbol{x}) = c$, the gradient vector $\nabla_x f(\boldsymbol{x})$ must also be normal to the same displacement $\Delta\boldsymbol{x}$ lying on the constraint surface:

$$\nabla_x f(\boldsymbol{x}).\Delta\boldsymbol{x} = 0 \quad (24)$$

At the stationary point, the two gradient vectors must be parallel and related by an unknown multiplier λ:

$$\{\nabla_x f(\boldsymbol{x}) + \lambda\nabla_x g(\boldsymbol{x})\}.\Delta\boldsymbol{x} = 0 \; for \; all \; \Delta\boldsymbol{x} \quad (25)$$

$$\Rightarrow \nabla_x f(\boldsymbol{x}) = -\lambda\nabla_x g(\boldsymbol{x}) \quad (26)$$

Equation (26) and the constraint $g(\boldsymbol{x}) = c$ are most conveniently satisfied by searching for stationary points of the Lagrangian function $F(\boldsymbol{x}, \lambda)$ defined in the $N + 1$ dimensional space comprising $\boldsymbol{x}$ and λ as:

$$F(\boldsymbol{x}, \lambda) = f(\boldsymbol{x}) + \lambda(g(\boldsymbol{x}) - c) \quad (27)$$

Setting all $N + 1$ components of the gradient of $F(\boldsymbol{x}, \lambda)$ to zero gives:

$$\nabla_{x,\lambda} F(\boldsymbol{x}, \lambda) = \nabla_{x,\lambda}\big(f(\boldsymbol{x}) + \lambda(g(\boldsymbol{x}) - c)\big) = 0 \quad (28)$$

Since λ is independent of $\boldsymbol{x}$ and c is constant, the first N derivatives with respect to $\boldsymbol{x}$ give the required constraint of equation (26):

$$\nabla_x F(\boldsymbol{x}, \lambda) = 0 \Rightarrow \nabla_x f(\boldsymbol{x}) + \lambda\nabla_x(g(\boldsymbol{x}) - c) = 0 \quad (29)$$

$$\Rightarrow \nabla_x f(\boldsymbol{x}) = -\lambda\nabla_x g(\boldsymbol{x}) \quad (30)$$

Since $f(\boldsymbol{x})$, $g(\boldsymbol{x})$ are independent of λ and c is constant, the last component of the derivative with respect to λ ensures that the constraint $g(\boldsymbol{x}) = c$ is satisfied:

$$\frac{\partial F(\boldsymbol{x}, \lambda)}{\partial\lambda} = 0 \Rightarrow \frac{\partial\{f(\boldsymbol{x}) + \lambda(g(\boldsymbol{x}) - c)\}}{\partial\lambda} = \frac{\partial\{\lambda(g(\boldsymbol{x}) - c)\}}{\partial\lambda} = g(\boldsymbol{x}) - c = 0 \quad (31)$$

2.4 Eigenvector Solution in Homogeneous Case

If the dependent measurement vector $\boldsymbol{Y}$ is zero, then the equations are termed homogeneous. The model being fitted in a least squares sense is now:

$$\boldsymbol{X\beta} = 0 \quad (32)$$

The error function E to be minimized simplifies to:

$$E(\boldsymbol{\beta}) = \|\boldsymbol{r}\|^2 = \|\boldsymbol{X\beta}\|^2 = (\boldsymbol{X\beta})^T\boldsymbol{X\beta} = \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} \quad (33)$$

Unfortunately, using the Normal Equations solution given by equation (16) for the non-homogeneous case gives the zero vector solution for $\boldsymbol{\beta}$ when $\boldsymbol{Y}$ is the null vector.

$$\boldsymbol{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y} = 0 \quad (34)$$

This is a valid solution but not terribly useful. A solution method is required that minimizes the error function $E(\boldsymbol{\beta})$ in equation (33) subject to the constraint that $\boldsymbol{\beta}$ has non-zero magnitude. Because equation (32) is linear, the solution vector $\boldsymbol{\beta}$ can be constrained to have unit magnitude:

$$1 - \boldsymbol{\beta}^T\boldsymbol{\beta} = 0 \quad (35)$$

Using the method of Lagrange Multipliers, the modified error function can be rewritten as:

$$E(\boldsymbol{\beta}) = (\boldsymbol{X\beta})^T\boldsymbol{X\beta} + \lambda(1 - \boldsymbol{\beta}^T\boldsymbol{\beta}) = \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} + \lambda(1 - \boldsymbol{\beta}^T\boldsymbol{\beta}) \quad (36)$$

Applying the stationary constraint that $E(\boldsymbol{\beta} + \delta\boldsymbol{\beta}) = E(\boldsymbol{\beta})$ to equation (36) gives:

$$(\boldsymbol{\beta} + \delta\boldsymbol{\beta})^T\boldsymbol{X}^T\boldsymbol{X}(\boldsymbol{\beta} + \delta\boldsymbol{\beta}) + \lambda\left(1 - (\boldsymbol{\beta} + \delta\boldsymbol{\beta})^T(\boldsymbol{\beta} + \delta\boldsymbol{\beta})\right) = \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} + \lambda(1 - \boldsymbol{\beta}^T\boldsymbol{\beta}) \quad (37)$$

Ignoring second order terms gives:

$$\delta\boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X\beta} + \boldsymbol{\beta}^T\boldsymbol{X}^T\boldsymbol{X}\delta\boldsymbol{\beta} - \lambda\boldsymbol{\beta}^T\delta\boldsymbol{\beta} - \lambda\delta\boldsymbol{\beta}^T\boldsymbol{\beta} = 0 \quad (38)$$

Because each term in equation (38) is a scalar and equal to its transpose, the solution for the optimum $\boldsymbol{\beta}$ which constrains the performance function is:

$$2\delta\boldsymbol{\beta}^T(\boldsymbol{X}^T\boldsymbol{X\beta} - \lambda\boldsymbol{\beta}) = 0 \quad (39)$$

$$\Rightarrow \boldsymbol{X}^T\boldsymbol{X\beta} = \lambda\boldsymbol{\beta} \quad (40)$$

Equation (40) states that the required solution vector $\boldsymbol{\beta}$ is one of the eigenvectors of the product matrix $\boldsymbol{X}^T\boldsymbol{X}$ with eigenvalue λ.

Substituting equation (40) into equation (33) gives the error function $E(\boldsymbol{\beta}_i)$ associated with eigenvalue λ_i and eigenvector $\boldsymbol{\beta}_i$ as:

$$E(\boldsymbol{\beta}_i) = \boldsymbol{\beta}_i{}^T\boldsymbol{X}^T\boldsymbol{X\beta}_i = \lambda_i\boldsymbol{\beta}_i{}^T\boldsymbol{\beta}_i = \lambda_i \quad (41)$$

The error function $E(\boldsymbol{\beta}_i)$ for the i^{th} eigenvector $\boldsymbol{\beta}_i$ therefore equals the associated eigenvalue λ_i. The minimum error function is therefore equal to the smallest eigenvalue λ_{min} of $\boldsymbol{X}^T\boldsymbol{X}$ and the required solution $\boldsymbol{\beta}_{min}$ is the eigenvector with the smallest eigenvalue.

2.5 Eigenvectors and Eigenvalues of Symmetric Matrices

The measurement matrix $\boldsymbol{X}^T\boldsymbol{X}$ is symmetric because:

$$(\boldsymbol{X}^T\boldsymbol{X})^T = \boldsymbol{X}^T\boldsymbol{X} \tag{42}$$

The eigenvectors $\boldsymbol{\beta}$ of $\boldsymbol{X}^T\boldsymbol{X}$ satisfy:

$$(\boldsymbol{\beta}_j)^T\boldsymbol{X}^T\boldsymbol{X}\,\boldsymbol{\beta}_k = \lambda_k(\boldsymbol{\beta}_j)^T\boldsymbol{\beta}_k \tag{43}$$

Transposing equation (43) gives:

$$(\boldsymbol{\beta}_k)^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{\beta}_j = \lambda_k(\boldsymbol{\beta}_k)^T\boldsymbol{\beta}_j \tag{44}$$

$$\Rightarrow \lambda_j(\boldsymbol{\beta}_k)^T\boldsymbol{\beta}_j = \lambda_k(\boldsymbol{\beta}_k)^T\boldsymbol{\beta}_j \tag{45}$$

$$\Rightarrow (\lambda_j - \lambda_k)(\boldsymbol{\beta}_k)^T\boldsymbol{\beta}_j = 0 \tag{46}$$

$$\Rightarrow (\boldsymbol{\beta}_k)^T\boldsymbol{\beta}_j = 0 \; if \; \lambda_j \neq \lambda_k \tag{47}$$

The eigenvectors of a symmetric matrix are therefore orthogonal if the eigenvalues are distinct.

The definition of a positive semi-definite matrix $\boldsymbol{A}$ is one that satisfies for all nonzero vectors $\boldsymbol{v}_j$:

$$(\boldsymbol{v}_j)^T\boldsymbol{A}\boldsymbol{v}_j \geq 0 \tag{48}$$

Setting $k = j$ in equation (43) gives:

$$(\boldsymbol{\beta}_j)^T\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{\beta}_j = (\boldsymbol{X}\boldsymbol{\beta}_j)^T\boldsymbol{X}\boldsymbol{\beta}_j = \lambda_j(\boldsymbol{\beta}_j)^T\boldsymbol{\beta}_j \tag{49}$$

$$\Rightarrow |\boldsymbol{X}\boldsymbol{\beta}_j|^2 = \lambda_j|\boldsymbol{\beta}_j|^2 \tag{50}$$

The left-hand side of equation (50) is non-negative. For nonzero $\boldsymbol{\beta}_j$ it therefore follows that the symmetric matrix $\boldsymbol{X}^T\boldsymbol{X}$ is positive semi-definite and has non-negative eigenvalues if the associated eigenvector has non-zero norm.

3. Hard and Soft Iron Magnetic Model

3.1 General Linear Model

The most general linear model for the *k-th* magnetometer measurement ${}^{S}\boldsymbol{B}_{k}$ in the sensor frame, as a consequence of hard and soft iron distortion to the true magnetic field ${}^{S}\boldsymbol{B}_{c,k}$ incident on the magnetometer, is:

$${}^{S}\boldsymbol{B}_{k} = \boldsymbol{W}\,{}^{S}\boldsymbol{B}_{c,k} + \boldsymbol{V} \tag{51}$$

The 3x1 vector $\boldsymbol{V}$ is termed the *hard iron offset* and the 3x3 matrix $\boldsymbol{W}$ is termed the *soft iron matrix*. The hard iron offset models the sensor's intrinsic zero magnetic field offset plus the effects of permanently magnetized components on the circuit board. The soft iron matrix models the directional effect of induced magnetic fields and differing sensitivities in the three axes of the magnetometer sensor.

The calibration algorithms derived in this document estimate the hard iron offset $\boldsymbol{V}$ and the soft iron matrix $\boldsymbol{W}$ from magnetometer measurements and then invert equation (51) to give the calibrated magnetometer measurement in the sensor frame ${}^{S}\boldsymbol{B}_{c,k}$ as:

$${}^{S}\boldsymbol{B}_{c,k} = \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right) \tag{52}$$

Equation (52) is implemented in the function `fInvertMagCal`.

In the absence of any extraneous magnetic disturbance, such as an external magnet, the true applied field in the sensor frame is simply the earth's geomagnetic field ${}^{G}\boldsymbol{B}_{0}$ rotated by the orientation matrix $\boldsymbol{R}$ defining the orientation of the magnetometer:

$${}^{S}\boldsymbol{B}_{c,k} = \boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} \tag{53}$$

The geomagnetic vector ${}^{G}\boldsymbol{B}_{0}$ is a constant vector in the global reference frame (pointing northwards and downwards in the northern hemisphere). The multiplication by the circuit board orientation matrix $\boldsymbol{R}$ is an example of a vector transformation from the global coordinate frame to the sensor coordinate frame.

Substituting equation (53) into equation (51) gives the model for the magnetometer measurement in terms of the rotated and distorted geomagnetic field vector as:

$${}^{S}\boldsymbol{B}_{k} = \boldsymbol{W}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} + \boldsymbol{V} \tag{54}$$

If the hard and soft iron calibration is accurately determined then the calibrated magnetometer measurement ${}^{S}\boldsymbol{B}_{c,k}$ is the simply the geomagnetic field rotated into the sensor frame:

$${}^{S}\boldsymbol{B}_{c,k} = \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right) = \boldsymbol{W}^{-1}\left\{\left(\boldsymbol{W}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} + \boldsymbol{V}\right) - \boldsymbol{V}\right\} = \boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} \tag{55}$$

3.2 Measurement Loci

Under arbitrary rotation of the magnetometer sensor, the locus of the calibrated magnetometer readings ${}^{S}\boldsymbol{B}_{c,k}$ satisfies:

$$\left|{}^{S}\boldsymbol{B}_{c,k}\right|^{2} = \left({}^{S}\boldsymbol{B}_{c,k}\right)^{T}\,{}^{S}\boldsymbol{B}_{c,k} = \left(\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0}\right)^{T}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} = \left({}^{G}\boldsymbol{B}_{0}\right)^{T}\boldsymbol{R}^{T}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} = \left|{}^{G}\boldsymbol{B}_{0}\right|^{2} = B^{2} \tag{56}$$

where B is the magnitude of the geomagnetic field vector ${}^{G}\boldsymbol{B}_{0}$. The calibrated measurements ${}^{S}\boldsymbol{B}_{c,k}$ therefore lie on the surface of the sphere centered at the origin with radius B.

The locus of the uncalibrated magnetometer readings ${}^{S}\boldsymbol{B}_{k}$ satisfies:

$$\left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right)\right\}^{T}\left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right)\right\} = \left(\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0}\right)^{T}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} = \left({}^{G}\boldsymbol{B}_{0}\right)^{T}\boldsymbol{R}^{T}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} = \left|{}^{G}\boldsymbol{B}_{0}\right|^{2} = B^{2} \tag{57}$$

$$\Rightarrow \left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right)^{T}(\boldsymbol{W}^{-1})^{T}\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_{k} - \boldsymbol{V}\right) = B^{2} \tag{58}$$

The general expression for the locus of a vector $\boldsymbol{u}$ lying on the surface of an ellipsoid with center at $\boldsymbol{u}_{0}$ is known to be:

$$(\boldsymbol{u} - \boldsymbol{u}_{0})^{T}\boldsymbol{A}(\boldsymbol{u} - \boldsymbol{u}_{0}) = const \tag{59}$$

where $\boldsymbol{A}$ is a symmetric matrix defining the shape of the ellipsoid.

Equations (58) and (59) are clearly of the same form with $\boldsymbol{A} = (\boldsymbol{W}^{-1})^{T}\boldsymbol{W}^{-1}$. It is easily proven that the matrix $\boldsymbol{A} = \{\boldsymbol{W}^{-1}\}^{T}\boldsymbol{W}^{-1}$ is symmetric:

$$\boldsymbol{A}^{T} = \{\{\boldsymbol{W}^{-1}\}^{T}\boldsymbol{W}^{-1}\}^{T} = \{\boldsymbol{W}^{-1}\}^{T}\{\{\boldsymbol{W}^{-1}\}^{T}\}^{T} = \{\boldsymbol{W}^{-1}\}^{T}\boldsymbol{W}^{-1} = \boldsymbol{A} \tag{60}$$

In the absence of any hard iron and soft iron distortion, the magnetometer measurements in the sensor frame lie on the surface of a sphere with radius equal to the geomagnetic field strength B. The hard iron offset vector $\boldsymbol{V}$ moves the center of the sphere to $\boldsymbol{V}$ and the soft iron matrix $\boldsymbol{W}$ distorts the sphere into an ellipsoid.

The calibration mapping of equation (52) transforms uncalibrated measurements ${}^{S}\boldsymbol{B}_{k}$ from their locus on the surface on the measurement ellipsoid onto the locus of the calibrated measurements ${}^{S}\boldsymbol{B}_{c,k}$ which is the surface of a sphere. The transformation has two components: i) subtraction of the hard iron vector $\boldsymbol{V}$ which centers the measurements at the origin and ii) multiplication by the inverse soft iron gain matrix $\boldsymbol{W}^{-1}$ which removes the ellipsoidal distortion.

3.3 Example Calibration Surfaces

Fig 1 shows measurements taken from a sensor circuit board with minimal ferromagnetic components. The uncalibrated measurements are shown in red and the calibrated measurements in blue. The soft iron matrix $\boldsymbol{W}$ is close to the identity matrix and the hard iron vector is dominated by a 100 µT offset in the z-axis which in this case results from the sensor's zero field offset. This type of circuit board could be calibrated for hard iron offset only, ignoring soft iron calibration, using the algorithm described in Section 4. The more sophisticated soft iron algorithms of Sections 5 and 6 could also be used but would provide little, or no, performance improvement.

In this particular example, the calibration mapping is approximately the translation:

$$ {}^{S}\boldsymbol{B}_{c,k} \approx {}^{S}\boldsymbol{B}_{k} - V_z\widehat{\boldsymbol{k}} \approx {}^{S}\boldsymbol{B}_{k} - 100\mu T\widehat{\boldsymbol{k}} \quad (61) $$

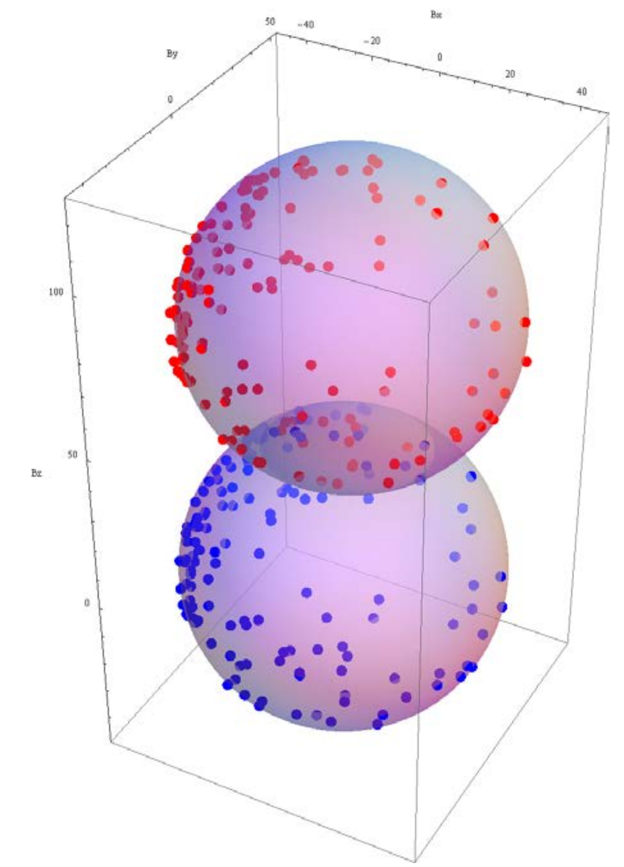

Fig 1. Uncalibrated (red) and calibrated (blue) measurements from a simple hard iron environment

Fig 2 shows measurements taken from an Android tablet with strong hard and soft iron distortions resulting from a ferromagnetic film behind the display. This type of distortion must be calibrated using the more sophisticated hard and soft iron algorithms described in Sections 5 and 6.

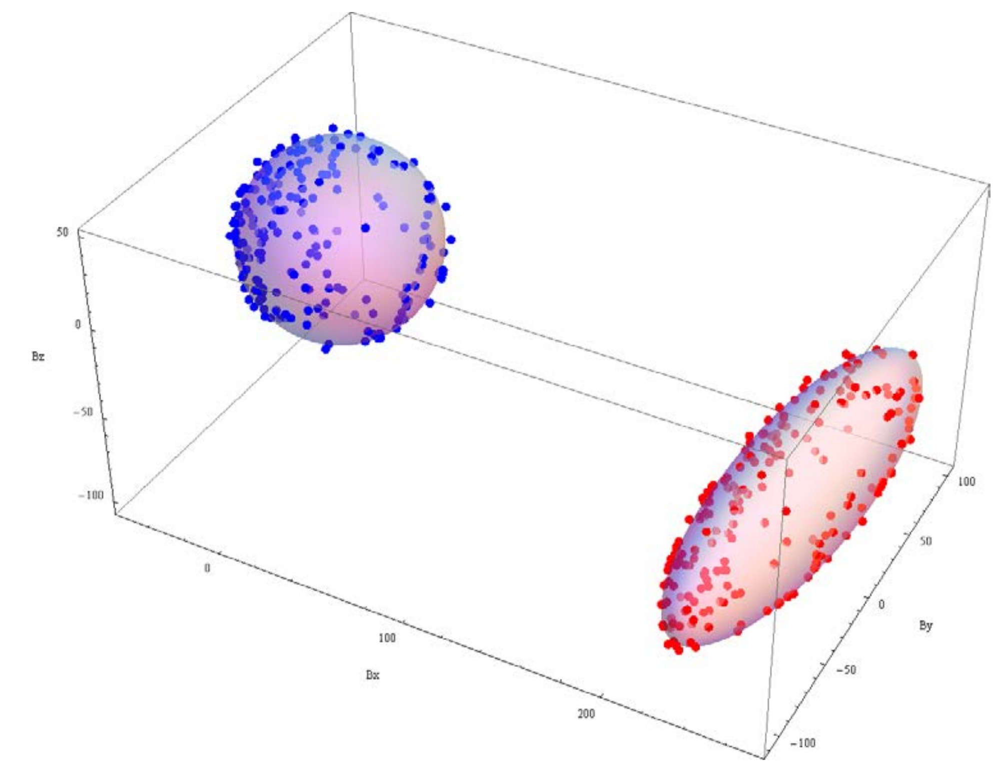

Fig 2. Uncalibrated (red) and calibrated (blue) measurements from a complex hard and soft iron environment

4. Four Parameter Magnetic Calibration Model

4.1 Derivation of the Least Squares Solution

This section documents the simplest magnetic calibration algorithm implemented in function `fUpdateMagCalibration4Slice` which calculates the four parameters comprising the hard iron offset vector $\boldsymbol{V}$ and geomagnetic field strength B. The soft iron matrix $\boldsymbol{W}$ in this model is the identity matrix $\boldsymbol{I}$. This model provides reasonable performance with high simplicity on simple circuit boards with low soft iron distortion.

The model for the magnetometer measurement ${}^{S}\boldsymbol{B}_k$ in terms of the true calibrated measurement ${}^{S}\boldsymbol{B}_{c,k}$, which is the geomagnetic field vector $\boldsymbol{B}_0$ rotated from the global to the sensor frame by rotation matrix $\boldsymbol{R}$, is:

$$ {}^{S}\boldsymbol{B}_k = {}^{S}\boldsymbol{B}_{c,k} + \boldsymbol{V} = \boldsymbol{R}\,{}^{G}\boldsymbol{B}_0 + \boldsymbol{V} \tag{62} $$

The rotation matrix $\boldsymbol{R}$ can be eliminated from equation (62) giving the measurement locus:

$$ \left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)^T\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right) = B^2 \Rightarrow {}^{S}\boldsymbol{B}_k{}^T\,{}^{S}\boldsymbol{B}_k - 2\,{}^{S}\boldsymbol{B}_k{}^T\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{V} = B^2 \tag{63} $$

Equation (63) models the locus of the magnetometer measurements ${}^{S}\boldsymbol{B}_k$ as lying on the surface of a sphere with radius B offset from the origin by $\boldsymbol{V}$.

The residual error r_k for the k^{th} magnetometer measurement is defined in terms of the deviation of the squared calibrated measurement from the squared geomagnetic field strength as:

$$ r_k = \left|{}^{S}\boldsymbol{B}_{c,k}\right|^2 - B^2 = \left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)^T\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right) - B^2 = {}^{S}\boldsymbol{B}_k{}^T\,{}^{S}\boldsymbol{B}_k - 2\,{}^{S}\boldsymbol{B}_k{}^T\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{V} - B^2 \tag{64} $$

The residual r_k therefore has dimensions of B^2. Expanding the components of equation (64) gives:

$$ r_k = {}^{S}B_{x,k}{}^2 + {}^{S}B_{y,k}{}^2 + {}^{S}B_{z,k}{}^2 - 2\,{}^{S}B_{x,k}V_x - 2\,{}^{S}B_{y,k}V_y - 2\,{}^{S}B_{z,k}V_z + V_x{}^2 + V_y{}^2 + V_z{}^2 - B^2 \tag{65} $$

Simplifying and returning to matrix format gives:

$$ r_k = \left({}^{S}B_{x,k}{}^2 + {}^{S}B_{y,k}{}^2 + {}^{S}B_{z,k}{}^2\right) - \begin{pmatrix} {}^{S}B_{x,k} & {}^{S}B_{y,k} & {}^{S}B_{z,k} & 1 \end{pmatrix} \begin{pmatrix} 2V_x \\ 2V_y \\ 2V_z \\ B^2 - V_x{}^2 - V_y{}^2 - V_z{}^2 \end{pmatrix} \tag{66} $$

Equation (66) can be expanded to represent M measurements as:

$$\begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{M-1} \end{pmatrix} = \begin{pmatrix} {^S}B_{x,0}{}^2 + {^S}B_{y,0}{}^2 + {^S}B_{z,0}{}^2 \\ {^S}B_{x,1}{}^2 + {^S}B_{y,1}{}^2 + {^S}B_{z,1}{}^2 \\ \dots \\ {^S}B_{x,M-1}{}^2 + {^S}B_{y,M-1}{}^2 + {^S}B_{z,M-1}{}^2 \end{pmatrix} - \begin{pmatrix} {^S}B_{x,0} & {^S}B_{y,0} & {^S}B_{z,0} & 1 \\ {^S}B_{x,1} & {^S}B_{y,1} & {^S}B_{z,1} & 1 \\ \dots & \dots & \dots & 1 \\ {^S}B_{x,M-1} & {^S}B_{y,M-1} & {^S}B_{z,M-1} & 1 \end{pmatrix} \boldsymbol{\beta} \quad (67)$$

where the 4×1 solution vector $\boldsymbol{\beta}$ is defined as:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 2V_x \\ 2V_y \\ 2V_z \\ B^2 - V_x{}^2 - V_y{}^2 - V_z{}^2 \end{pmatrix} \quad (68)$$

With the definitions of the $M \times 1$ error residual vector $\boldsymbol{r}$ as:

$$\boldsymbol{r} = \begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{M-1} \end{pmatrix} \quad (69)$$

and $\boldsymbol{Y}$ the $M \times 1$ vector of dependent variables:

$$\boldsymbol{Y} = \begin{pmatrix} {^S}B_{x,0}{}^2 + {^S}B_{y,0}{}^2 + {^S}B_{z,0}{}^2 \\ {^S}B_{x,1}{}^2 + {^S}B_{y,1}{}^2 + {^S}B_{z,1}{}^2 \\ \dots \\ {^S}B_{x,M-1}{}^2 + {^S}B_{y,M-1}{}^2 + {^S}B_{z,M-1}{}^2 \end{pmatrix} \quad (70)$$

and $\boldsymbol{X}$ the $M \times 4$ measurement matrix:

$$\boldsymbol{X} = \begin{pmatrix} {^S}B_{x,0} & {^S}B_{y,0} & {^S}B_{z,0} & 1 \\ {^S}B_{x,1} & {^S}B_{y,1} & {^S}B_{z,1} & 1 \\ \dots & \dots & \dots & 1 \\ {^S}B_{x,M-1} & {^S}B_{y,M-1} & {^S}B_{z,M-1} & 1 \end{pmatrix} \quad (71)$$

then equation (67) can be written as:

$$\boldsymbol{r} = \boldsymbol{Y} - \boldsymbol{X\beta} \quad (72)$$

The model being fitted has the non-homogeneous form $\boldsymbol{r} = \boldsymbol{Y} - \boldsymbol{X\beta}$ and can be solved using the Normal Equations method documented in Section 2.

The matrices $\boldsymbol{X}^T\boldsymbol{X}$, $\boldsymbol{X}^T\boldsymbol{Y}$ and $\boldsymbol{Y}^T\boldsymbol{Y}$ expand to:

$$\boldsymbol{X}^T\boldsymbol{X} = \begin{pmatrix} {}^SB_{x,0} & {}^SB_{y,0} & {}^SB_{z,0} & 1 \\ {}^SB_{x,1} & {}^SB_{y,1} & {}^SB_{z,1} & 1 \\ \dots & \dots & \dots & 1 \\ {}^SB_{x,M-1} & {}^SB_{y,M-1} & {}^SB_{z,M-1} & 1 \end{pmatrix}^T \begin{pmatrix} {}^SB_{x,0} & {}^SB_{y,0} & {}^SB_{z,0} & 1 \\ {}^SB_{x,1} & {}^SB_{y,1} & {}^SB_{z,1} & 1 \\ \dots & \dots & \dots & 1 \\ {}^SB_{x,M-1} & {}^SB_{y,M-1} & {}^SB_{z,M-1} & 1 \end{pmatrix} \tag{73}$$

$$\boldsymbol{X}^T\boldsymbol{X} = \sum_{k=0}^{M-1} \begin{pmatrix} {}^SB_{x,k}{}^2 & {}^SB_{x,k}\,{}^SB_{y,k} & {}^SB_{x,k}\,{}^SB_{z,k} & {}^SB_{x,k} \\ {}^SB_{x,k}\,{}^SB_{y,k} & {}^SB_{y,k}{}^2 & {}^SB_{y,k}\,{}^SB_{z,k} & {}^SB_{y,k} \\ {}^SB_{x,k}\,{}^SB_{z,k} & {}^SB_{y,k}\,{}^SB_{z,k} & {}^SB_{z,k}{}^2 & {}^SB_{z,k} \\ {}^SB_{x,k} & {}^SB_{y,k} & {}^SB_{z,k} & 1 \end{pmatrix} \tag{74}$$

$$\boldsymbol{X}^T\boldsymbol{Y} = \begin{pmatrix} {}^SB_{x,0} & {}^SB_{y,0} & {}^SB_{z,0} & 1 \\ {}^SB_{x,1} & {}^SB_{y,1} & {}^SB_{z,1} & 1 \\ \dots & \dots & \dots & 1 \\ {}^SB_{x,M-1} & {}^SB_{y,M-1} & {}^SB_{z,M-1} & 1 \end{pmatrix}^T \begin{pmatrix} {}^SB_{x,0}{}^2 + {}^SB_{y,0}{}^2 + {}^SB_{z,0}{}^2 \\ {}^SB_{x,1}{}^2 + {}^SB_{y,1}{}^2 + {}^SB_{z,1}{}^2 \\ \dots \\ {}^SB_{x,M-1}{}^2 + {}^SB_{y,M-1}{}^2 + {}^SB_{z,M-1}{}^2 \end{pmatrix} \tag{75}$$

$$= \sum_{k=0}^{M-1} \begin{pmatrix} {}^SB_{x,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ {}^SB_{y,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ {}^SB_{z,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ \left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \end{pmatrix} \tag{76}$$

$$\boldsymbol{Y}^T\boldsymbol{Y} = \begin{pmatrix} {}^SB_{x,0}{}^2 + {}^SB_{y,0}{}^2 + {}^SB_{z,0}{}^2 \\ {}^SB_{x,1}{}^2 + {}^SB_{y,1}{}^2 + {}^SB_{z,1}{}^2 \\ \dots \\ {}^SB_{x,M-1}{}^2 + {}^SB_{y,M-1}{}^2 + {}^SB_{z,M-1}{}^2 \end{pmatrix}^T \begin{pmatrix} {}^SB_{x,0}{}^2 + {}^SB_{y,0}{}^2 + {}^SB_{z,0}{}^2 \\ {}^SB_{x,1}{}^2 + {}^SB_{y,1}{}^2 + {}^SB_{z,1}{}^2 \\ \dots \\ {}^SB_{x,M-1}{}^2 + {}^SB_{y,M-1}{}^2 + {}^SB_{z,M-1}{}^2 \end{pmatrix} \tag{77}$$

$$= \sum_{k=0}^{M-1} \left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right)^2 \tag{78}$$

The solution vector $\boldsymbol{\beta}$ is then given by equation (16) as:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \left\{ \sum_{k=0}^{M-1} \begin{pmatrix} {}^SB_{x,k}{}^2 & {}^SB_{x,k}\,{}^SB_{y,k} & {}^SB_{x,k}\,{}^SB_{z,k} & {}^SB_{x,k} \\ {}^SB_{x,k}\,{}^SB_{y,k} & {}^SB_{y,k}{}^2 & {}^SB_{y,k}\,{}^SB_{z,k} & {}^SB_{y,k} \\ {}^SB_{x,k}\,{}^SB_{z,k} & {}^SB_{y,k}\,{}^SB_{z,k} & {}^SB_{z,k}{}^2 & {}^SB_{z,k} \\ {}^SB_{x,k} & {}^SB_{y,k} & {}^SB_{z,k} & 1 \end{pmatrix} \right\}^{-1} \sum_{k=0}^{M-1} \begin{pmatrix} {}^SB_{x,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ {}^SB_{y,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ {}^SB_{z,k}\left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \\ \left({}^SB_{x,k}{}^2 + {}^SB_{y,k}{}^2 + {}^SB_{z,k}{}^2\right) \end{pmatrix} \tag{79}$$

4.2 Hard Iron Vector

The hard iron solution vector is given directly by equation (68) as:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \left(\frac{1}{2}\right)\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} \tag{80}$$

4.3 Geomagnetic Field Strength

The geomagnetic field strength is computed from the last component of equation (68) as:

$$B^2 = \beta_3 + {V_x}^2 + {V_y}^2 + {V_z}^2 \Rightarrow B = \sqrt{\beta_3 + {V_x}^2 + {V_y}^2 + {V_z}^2} \tag{81}$$

4.4 Fit Error

The residuals r_k have dimensions of the geomagnetic field strength squared or B^2. The error function E is proportional to the sum of the M squared residuals and has dimensions of B^4. A dimensionless measure of fit error ε, independent of the number of measurements, is:

$$\varepsilon = \frac{1}{2}\sqrt{\frac{E(\boldsymbol{\beta})}{MB^4}} = \frac{1}{2B^2}\sqrt{\frac{E(\boldsymbol{\beta})}{M}} \tag{82}$$

The scaling factor of 2 is for mathematical convenience and has no particular significance.

The sensor fusion software returns the normalized fit error as the percentage $\varepsilon\%$ defined as:

$$\varepsilon\% = \frac{50}{B^2}\sqrt{\frac{E(\boldsymbol{\beta})}{M}} \tag{83}$$

5. Seven Parameter Magnetic Calibration Model

5.1 Derivation of the Least Squares Solution

This section documents the magnetic calibration algorithm implemented in function `fUpdateMagCalibration7Slice` which extends the four-parameter model of the previous section with the addition of three gain terms on the diagonal of the soft iron matrix $\boldsymbol{W}$ giving a total of seven magnetic calibration parameters. This model gives a significant improvement when either the magnetometer sensor has differing gains in its three channels or when the PCB has differing magnetic impedances along its three Cartesian axes. The diagonal form of $\boldsymbol{W}$ means that the magnetic distribution ellipsoid is modeled as having its principal axes aligned with the PCB's Cartesian axes. If this is not the case then the ten-parameter calibration model of Section 6 should be used instead.

The model for the magnetometer measurement ${}^{S}\boldsymbol{B}_k$ in terms of the true calibrated measurement ${}^{S}\boldsymbol{B}_{c,k}$, which is the geomagnetic field vector $\boldsymbol{B}_0$ rotated from the global to the sensor frame by rotation matrix $\boldsymbol{R}$, is:

$$ {}^{S}\boldsymbol{B}_k = \boldsymbol{W}\,{}^{S}\boldsymbol{B}_{c,k} + \boldsymbol{V} = \boldsymbol{W}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_0 + \boldsymbol{V} \tag{84} $$

In the seven-parameter magnetic calibration model, the soft iron matrix $\boldsymbol{W}$ is diagonal but not, in general, equal to the identity matrix.

The rotation matrix $\boldsymbol{R}$ can be eliminated from equation (84) giving the measurement locus:

$$ \left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)\right\}^T \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right) = \left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)^T (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right) = B^2 \tag{85} $$

$$ \Rightarrow \left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)^T \boldsymbol{A}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right) = B^2 \tag{86} $$

Equation (86) models the locus of the magnetometer measurements ${}^{S}\boldsymbol{B}_k$ as lying on the surface of an ellipsoid offset from the origin by $\boldsymbol{V}$ with axes defined by the ellipsoid matrix $\boldsymbol{A} = (\boldsymbol{W}^{-1})^T \boldsymbol{W}^{-1}$. Because $\boldsymbol{A}$ is diagonal, the ellipsoid's axes are aligned with the coordinate system's Cartesian axes.

The manipulations that follow derive an expression for the error residual r_k defined, for the k^{th} magnetometer measurement, in terms of the deviation of the squared calibrated measurement from the geomagnetic sphere as:

$$ r_k = \left|{}^{S}\boldsymbol{B}_{c,k}\right|^2 - B^2 = \left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)\right\}^T \left\{\boldsymbol{W}^{-1}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)\right\} - B^2 \tag{87} $$

$$ = \left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)^T \boldsymbol{A}\left({}^{S}\boldsymbol{B}_k - \boldsymbol{V}\right)\Big\} - B^2 \tag{88} $$

r_k has dimensions of B^2 in the same manner as the definition of the error residual in the four-parameter calibration model.

Expanding equation (87) gives:

$$r_k = \left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{A}\, {}^S\boldsymbol{B}_k - \left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{AV} - \boldsymbol{V}^T \boldsymbol{A}\, {}^S\boldsymbol{B}_k + \boldsymbol{V}^T \boldsymbol{AV} - B^2 \tag{89}$$

The term $\left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{AV}$ is a scalar and therefore unchanged under transposition:

$$\left\{ \left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{AV} \right\}^T = \boldsymbol{V}^T \boldsymbol{A}\, {}^S\boldsymbol{B}_k = \left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{AV} \tag{90}$$

Substituting equation (90) into equation (89) and rearranging gives:

$$r_k = \left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{A}\, {}^S\boldsymbol{B}_k - 2\left({}^S\boldsymbol{B}_k \right)^T \boldsymbol{AV} + \boldsymbol{V}^T \boldsymbol{AV} - B^2 \tag{91}$$

Expanding equation (91) into its individual components gives:

$$\begin{aligned} r_k = {} & A_{xx}\, {}^SB_{x,k}{}^2 + A_{yy}\, {}^SB_{y,k}{}^2 + A_{zz}\, {}^SB_{z,k}{}^2 - 2\, {}^SB_{x,k} A_{xx} V_x - 2\, {}^SB_{y,k} A_{yy} V_y - 2\, {}^SB_{z,k} A_{zz} V_z + A_{xx} V_x{}^2 \\ & + A_{yy} V_y{}^2 + A_{zz} V_z{}^2 - B^2 \end{aligned} \tag{92}$$

Simplifying and returning to matrix format gives:

$$r_k = \begin{pmatrix} {}^SB_{x,k}{}^2 \\ {}^SB_{y,k}{}^2 \\ {}^SB_{z,k}{}^2 \\ {}^SB_{x,k} \\ {}^SB_{y,k} \\ {}^SB_{z,k} \\ 1 \end{pmatrix}^T \begin{pmatrix} A_{xx} \\ A_{yy} \\ A_{zz} \\ -2A_{xx}V_x \\ -2A_{yy}V_y \\ -2A_{zz}V_z \\ A_{xx}V_x{}^2 + A_{yy}V_y{}^2 + A_{zz}V_z{}^2 - B^2 \end{pmatrix} \tag{93}$$

With the definition of the right-hand side of equation (93) as the 7×1 solution vector $\boldsymbol{\beta}$:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix} = \begin{pmatrix} A_{xx} \\ A_{yy} \\ A_{zz} \\ -2A_{xx}V_x \\ -2A_{yy}V_y \\ -2A_{zz}V_z \\ A_{xx}V_x{}^2 + A_{yy}V_y{}^2 + A_{zz}V_z{}^2 - B^2 \end{pmatrix} \tag{94}$$

then equation (92) for the error residual r_k, whose squared sum is to be minimized, is:

$$r_k = \begin{pmatrix} {}^SB_{x,k}{}^2 \\ {}^SB_{y,k}{}^2 \\ {}^SB_{z,k}{}^2 \\ {}^SB_{x,k} \\ {}^SB_{y,k} \\ {}^SB_{z,k} \\ 1 \end{pmatrix}^T \boldsymbol{\beta} \tag{95}$$

The error residual vector $\boldsymbol{r}$ from M magnetometer measurements is defined as the $M \times 1$ vector:

$$\boldsymbol{r} = \begin{pmatrix} r_0 \\ r_1 \\ \dots \\ r_{M-1} \end{pmatrix} \tag{96}$$

The $M \times 7$ measurement matrix $\boldsymbol{X}$ containing the M measurements is defined as:

$$\boldsymbol{X} = \begin{pmatrix} {}^SB_{x,0}{}^2 & {}^SB_{y,0}{}^2 & {}^SB_{z,0}{}^2 & {}^SB_{x,0} & {}^SB_{y,0} & {}^SB_{z,0} & 1 \\ {}^SB_{x,1}{}^2 & {}^SB_{y,1}{}^2 & {}^SB_{z,1}{}^2 & {}^SB_{x,1} & {}^SB_{y,1} & {}^SB_{z,1} & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & 1 \\ {}^SB_{x,M-1}{}^2 & {}^SB_{y,M-1}{}^2 & {}^SB_{z,M-1}{}^2 & {}^SB_{x,M-1} & {}^SB_{y,M-1} & {}^SB_{z,M-1} & 1 \end{pmatrix} \tag{97}$$

With these definitions, equation (93) can be expanded to represent all M measurements as:

$$\boldsymbol{r} = \boldsymbol{X}\boldsymbol{\beta} \tag{98}$$

The error model being fitted is the homogeneous model $\boldsymbol{X}\boldsymbol{\beta} = 0$ which can be solved for $\boldsymbol{\beta}$ using the Lagrange Multiplier and eigen-decomposition approach described in Section 2.

The 7×7 product matrix $\boldsymbol{X}^T\boldsymbol{X}$ whose eigenvectors and eigenvalues are to be determined evaluates to:

$$\boldsymbol{X}^T\boldsymbol{X} = \begin{pmatrix} {}^SB_{x,0}{}^2 & {}^SB_{x,1}{}^2 & \dots & {}^SB_{x,M-1}{}^2 \\ {}^SB_{y,0}{}^2 & {}^SB_{y,1}{}^2 & \dots & {}^SB_{y,M-1}{}^2 \\ {}^SB_{z,0}{}^2 & {}^SB_{z,1}{}^2 & \dots & {}^SB_{z,M-1}{}^2 \\ {}^SB_{x,0} & {}^SB_{x,1} & \dots & {}^SB_{x,M-1} \\ {}^SB_{y,0} & {}^SB_{y,1} & \dots & {}^SB_{y,M-1} \\ {}^SB_{z,0} & {}^SB_{z,0} & \dots & {}^SB_{z,M-1} \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} {}^SB_{x,0}{}^2 & {}^SB_{y,0}{}^2 & {}^SB_{z,0}{}^2 & {}^SB_{x,0} & {}^SB_{y,0} & {}^SB_{z,0} & 1 \\ {}^SB_{x,1}{}^2 & {}^SB_{y,1}{}^2 & {}^SB_{z,1}{}^2 & {}^SB_{x,1} & {}^SB_{y,1} & {}^SB_{z,1} & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & 1 \\ {}^SB_{x,M-1}{}^2 & {}^SB_{y,M-1}{}^2 & {}^SB_{z,M-1}{}^2 & {}^SB_{x,M-1} & {}^SB_{y,M-1} & {}^SB_{z,M-1} & 1 \end{pmatrix} \tag{99}$$

$$= \sum_{k=0}^{M-1} \begin{pmatrix} {}^{s}B_{x,k}{}^{4} & {}^{s}B_{x,k}{}^{2}\,{}^{s}B_{y,k}{}^{2} & {}^{s}B_{x,k}{}^{2}\,{}^{s}B_{z,k}{}^{2} & {}^{s}B_{x,k}{}^{3} & {}^{s}B_{x,k}{}^{2}\,{}^{s}B_{y,k} & {}^{s}B_{x,k}{}^{2}\,{}^{s}B_{z,k} & {}^{s}B_{x,k}{}^{2} \\ {}^{s}B_{y,k}{}^{2}\,{}^{s}B_{x,k}{}^{2} & {}^{s}B_{y,k}{}^{4} & {}^{s}B_{y,k}{}^{2}\,{}^{s}B_{z,k}{}^{2} & {}^{s}B_{y,k}{}^{2}\,{}^{s}B_{x,k} & {}^{s}B_{y,k}{}^{3} & {}^{s}B_{y,k}{}^{2}\,{}^{s}B_{z,k} & {}^{s}B_{y,k}{}^{2} \\ {}^{s}B_{z,k}{}^{2}\,{}^{s}B_{x,k}{}^{2} & {}^{s}B_{z,k}{}^{2}\,{}^{s}B_{y,k}{}^{2} & {}^{s}B_{z,k}{}^{4} & {}^{s}B_{z,k}{}^{2}\,{}^{s}B_{x,k} & {}^{s}B_{z,k}{}^{2}\,{}^{s}B_{y,k} & {}^{s}B_{z,k}{}^{3} & {}^{s}B_{z,k}{}^{2} \\ {}^{s}B_{x,k}{}^{3} & {}^{s}B_{x,k}\,{}^{s}B_{y,k}{}^{2} & {}^{s}B_{x,k}\,{}^{s}B_{z,k}{}^{2} & {}^{s}B_{x,k}{}^{2} & {}^{s}B_{x,k}\,{}^{s}B_{y,k} & {}^{s}B_{x,k}\,{}^{s}B_{z,k} & {}^{s}B_{x,k} \\ {}^{s}B_{y,k}\,{}^{s}B_{x,k}{}^{2} & {}^{s}B_{y,k}{}^{3} & {}^{s}B_{y,k}\,{}^{s}B_{z,k}{}^{2} & {}^{s}B_{y,k}\,{}^{s}B_{x,k} & {}^{s}B_{y,k}{}^{2} & {}^{s}B_{y,k}\,{}^{s}B_{z,k} & {}^{s}B_{y,k} \\ {}^{s}B_{z,k}\,{}^{s}B_{x,k}{}^{2} & {}^{s}B_{z,k}\,{}^{s}B_{y,k}{}^{2} & {}^{s}B_{z,k}{}^{3} & {}^{s}B_{z,k}\,{}^{s}B_{x,k} & {}^{s}B_{z,k}\,{}^{s}B_{y,k} & {}^{s}B_{z,k}{}^{2} & {}^{s}B_{z,k} \\ {}^{s}B_{x,k}{}^{2} & {}^{s}B_{y,k}{}^{2} & {}^{s}B_{z,k}{}^{2} & {}^{s}B_{x,k} & {}^{s}B_{y,k} & {}^{s}B_{z,k} & 1 \end{pmatrix} \tag{100}$$

Because the eigenvalues of $\boldsymbol{X}^T\boldsymbol{X}$ are equal to the fit errors associated with the 7 candidate eigenvector solutions, the required solution vector $\boldsymbol{\beta}_{min}$ is the eigenvector associated with the smallest eigenvalue λ_{min}.

5.2 Ellipsoid Fit Matrix

The ellipsoid fit matrix $\boldsymbol{A}$ is obtained directly from the first three rows of the solution vector $\boldsymbol{\beta}$ in equation (94):

$$\boldsymbol{A} = \begin{pmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix} = \begin{pmatrix} \beta_0 & 0 & 0 \\ 0 & \beta_1 & 0 \\ 0 & 0 & \beta_2 \end{pmatrix} \tag{101}$$

The solution eigenvector $\boldsymbol{\beta}$ is undefined within a multiplicative factor of ±1 (assuming it is normalized to unit magnitude). Since physically sensible solutions for $\boldsymbol{A}$ require that it have a positive determinant, the entire solution vector $\boldsymbol{\beta}$ should therefore be negated if $|\boldsymbol{A}| < 0$.

The ellipsoid matrix $\boldsymbol{A}$ is normalized to have unit determinant:

$$|\boldsymbol{A}| = \begin{vmatrix} A_{xx} & 0 & 0 \\ 0 & A_{yy} & 0 \\ 0 & 0 & A_{zz} \end{vmatrix} = A_{xx}A_{yy}A_{zz} = 1 \tag{102}$$

The justification for the normalization in equation (102) is that it is physically impossible to separate out the magnitude of the geomagnetic field strength B from the soft iron matrix gain terms. A 25 µT geomagnetic field strength B with unit soft iron matrix gain results in the same magnetometer measurement as a 50 µT geomagnetic field strength B attenuated 50% by magnetic shielding. The solution taken in the NXP software always sets the determinant of the soft iron matrix $|\boldsymbol{A}| = 1$ and computes the geomagnetic field strength B on this assumption.

5.3 Hard Iron Vector

The hard iron vector $\boldsymbol{V}$ is given by equation (94) as:

$$\begin{pmatrix} -2A_{xx}V_x \\ -2A_{yy}V_y \\ -2A_{zz}V_z \end{pmatrix} = \begin{pmatrix} \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} \Rightarrow \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} \left(\frac{-\beta_3}{2A_{xx}}\right) \\ \left(\frac{-\beta_4}{2A_{yy}}\right) \\ \left(\frac{-\beta_5}{2A_{zz}}\right) \end{pmatrix} = \begin{pmatrix} \left(\frac{-\beta_3}{2\beta_0}\right) \\ \left(\frac{-\beta_4}{2\beta_1}\right) \\ \left(\frac{-\beta_5}{2\beta_2}\right) \end{pmatrix} \tag{103}$$

The hard iron vector $\boldsymbol{V}$ is obviously unchanged if the entire solution vector $\boldsymbol{\beta}$ is negated to force the ellipsoid matrix $\boldsymbol{A}$ to have positive determinant since the sign change occurs on both numerator and denominator.

5.4 Inverse Soft Iron Matrix

The inverse soft iron matrix can be found from the square root of the diagonal ellipsoid matrix as:

$$\boldsymbol{W}^{-1} = \begin{pmatrix} W_{xx} & 0 & 0 \\ 0 & W_{yy} & 0 \\ 0 & 0 & W_{zz} \end{pmatrix} = \sqrt{\boldsymbol{A}} = \begin{pmatrix} \sqrt{\beta_0} & 0 & 0 \\ 0 & \sqrt{\beta_1} & 0 \\ 0 & 0 & \sqrt{\beta_2} \end{pmatrix} \tag{104}$$

Physically sensible solutions have W_{xx}, W_{yy} and W_{zz} all positive. The case where the negative eigenvector is returned by the eigensolver is handled by computing the determinant of the ellipsoid matrix $\boldsymbol{A}$ and negating the entire solution vector $\boldsymbol{\beta}$ if negative. β_0, β_1 and β_2 will therefore be positive in equation (104) allowing the positive square root to be taken.

5.5 Geomagnetic Field Strength

The geomagnetic field strength B is given by the last component of equation (94):

$$\beta_6 = A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 - B^2 \Rightarrow B = \sqrt{A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 - \beta_6} \tag{105}$$

5.6 Fit Error

The error function E equals the smallest eigenvalue λ_{min} of the product matrix $\boldsymbol{X}^T\boldsymbol{X}$ but is not normalized to either the number of measurement points M nor to the geomagnetic field strength B. Because $E = \boldsymbol{r}^T\boldsymbol{r}$ and $\boldsymbol{r}$ has M elements with each element having dimensions B^2, a suitable normalized calibration fit error measurement ε is:

$$\varepsilon = \frac{1}{2B^2}\sqrt{\frac{\lambda_{min}}{M}} \quad \textbf{(106)}$$

The sensor fusion software returns the normalized fit error as the percentage $\varepsilon\%$ defined as:

$$\varepsilon\% = \frac{50}{B^2}\sqrt{\frac{\lambda_{min}}{M}} \quad \textbf{(107)}$$

6. Ten Parameter Magnetic Calibration Model

6.1 Derivation of the Least Squares Solution

This section documents the magnetic calibration algorithm implemented in function `fUpdateMagCalibration10Slice`, which extends the seven-parameter model of the previous section with the addition of three off-diagonal soft iron matrix terms to $\boldsymbol{W}$ to give a total of ten magnetic calibration parameters. This model gives an improvement over the seven-parameter model when the PCB's magnetic impedances steer the geomagnetic field in directions that are not aligned with the PCB's Cartesian axes giving a rotated magnetic ellipsoid.

The model for the magnetometer measurement ${}^{S}\boldsymbol{B}_{k}$ in terms of the true calibrated measurement ${}^{S}\boldsymbol{B}_{c,k}$, which is the geomagnetic field vector $\boldsymbol{B}_{0}$ rotated from the global to the sensor frame by rotation matrix $\boldsymbol{R}$, is:

$$ {}^{S}\boldsymbol{B}_{k} = \boldsymbol{W}\,{}^{S}\boldsymbol{B}_{c,k} + \boldsymbol{V} = \boldsymbol{W}\boldsymbol{R}\,{}^{G}\boldsymbol{B}_{0} + \boldsymbol{V} \tag{108} $$

In the ten-parameter magnetic calibration model, the soft iron matrix $\boldsymbol{W}$ is symmetric but otherwise unconstrained.

The locus of the magnetometer measurements is:

$$ \{\boldsymbol{W}^{-1}(\boldsymbol{B}_{s}-\boldsymbol{V})\}^{T}\boldsymbol{W}^{-1}(\boldsymbol{B}_{s}-\boldsymbol{V}) = (\boldsymbol{B}_{s}-\boldsymbol{V})^{T}(\boldsymbol{W}^{-1})^{T}\boldsymbol{W}^{-1}(\boldsymbol{B}_{s}-\boldsymbol{V}) = (\boldsymbol{B}_{s}-\boldsymbol{V})^{T}\boldsymbol{A}(\boldsymbol{B}_{s}-\boldsymbol{V}) = B^{2} \tag{109} $$

where $\boldsymbol{A} = (\boldsymbol{W}^{-1})^{T}\boldsymbol{W}^{-1}$.

Equation (109) models the locus of the magnetometer measurements $\boldsymbol{B}_{s}$ as lying on the surface of an ellipsoid with arbitrary dimensions and directions of its axes and offset from the origin by the hard iron vector $\boldsymbol{V}$.

The manipulations that follow derive an expression for the error residual r_{k}, defined in the same manner as for the 4 and 7 parameter models, as:

$$ r_{k} = \left|{}^{S}\boldsymbol{B}_{c,k}\right|^{2} - B^{2} = \left({}^{S}\boldsymbol{B}_{k}-\boldsymbol{V}\right)^{T}\boldsymbol{A}\left({}^{S}\boldsymbol{B}_{k}-\boldsymbol{V}\right)\} - B^{2} \tag{110} $$

$$ = \left({}^{S}\boldsymbol{B}_{k}\right)^{T}\boldsymbol{A}\,{}^{S}\boldsymbol{B}_{k} - 2\left({}^{S}\boldsymbol{B}_{k}\right)^{T}\boldsymbol{A}\boldsymbol{V} + \boldsymbol{V}^{T}\boldsymbol{A}\boldsymbol{V} - B^{2} \tag{111} $$

The only difference between equation (91) for the seven-parameter model and equation (111) for the ten-parameter model is that the ellipsoid matrix $\boldsymbol{A}$ is diagonal in equation (91) but is symmetric with off-diagonal terms in equation (111).

Expanding equation (111) gives:

$$r_k = \begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}$$
$$- 2\begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} + \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} - B^2 \tag{112}$$

The first term in equation (112) expands to:

$$\begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix} = \begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx}\,{}^{S}B_{x,k} + A_{xy}\,{}^{S}B_{y,k} + A_{xz}\,{}^{S}B_{z,k} \\ A_{xy}\,{}^{S}B_{x,k} + A_{yy}\,{}^{S}B_{y,k} + A_{yz}\,{}^{S}B_{z,k} \\ A_{xz}\,{}^{S}B_{x,k} + A_{yz}\,{}^{S}B_{y,k} + A_{zz}\,{}^{S}B_{z,k} \end{pmatrix} \tag{113}$$

$$= A_{xx}\,{}^{S}B_{x,k}{}^{2} + A_{yy}\,{}^{S}B_{y,k}{}^{2} + A_{zz}\,{}^{S}B_{z,k}{}^{2} + 2A_{xy}\,{}^{S}B_{x,k}\,{}^{S}B_{y,k} + 2A_{xz}\,{}^{S}B_{x,k}\,{}^{S}B_{z,k} + 2A_{yz}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} \tag{114}$$

The second term in equation (112) expands to:

$$-2\begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = -2\begin{pmatrix} {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \end{pmatrix}^T \begin{pmatrix} A_{xx}V_x + A_{xy}V_y + A_{xz}V_z \\ A_{xy}V_x + A_{yy}V_y + A_{yz}V_z \\ A_{xz}V_x + A_{yz}V_y + A_{zz}V_z \end{pmatrix} \tag{115}$$

$$\begin{aligned} = -2\,{}^{S}B_{x,k}A_{xx}V_x - 2\,{}^{S}B_{x,k}A_{xy}V_y - 2\,{}^{S}B_{x,k}A_{xz}V_z - 2\,{}^{S}B_{y,k}A_{xy}V_x - 2\,{}^{S}B_{y,k}A_{yy}V_y \\ - 2\,{}^{S}B_{y,k}A_{yz}V_z - 2\,{}^{S}B_{z,k}A_{xz}V_x - 2\,{}^{S}B_{z,k}A_{yz}V_y - 2\,{}^{S}B_{z,k}A_{zz}V_z \end{aligned} \tag{116}$$

The third term in equation (112) expands to:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^T \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^T \begin{pmatrix} A_{xx}V_x + A_{xy}V_y + A_{xz}V_z \\ A_{xy}V_x + A_{yy}V_y + A_{yz}V_z \\ A_{xz}V_x + A_{yz}V_y + A_{zz}V_z \end{pmatrix} \tag{117}$$

$$= A_{xx}{V_x}^2 + A_{yy}{V_y}^2 + A_{zz}{V_z}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + 2A_{yz}V_yV_z \tag{118}$$

The full equation for the residual error r_k from the *k-th* observation is then:

$$r_k = A_{xx}\,{}^{S}B_{x,k}{}^{2} + A_{yy}\,{}^{S}B_{y,k}{}^{2} + A_{zz}\,{}^{S}B_{z,k}{}^{2} + 2A_{xy}\,{}^{S}B_{x,k}\,{}^{S}B_{y,k} + 2A_{xz}\,{}^{S}B_{x,k}\,{}^{S}B_{z,k} + 2A_{yz}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k}$$

$$-2\,{}^{S}B_{x,k}A_{xx}V_x - 2\,{}^{S}B_{x,k}A_{xy}V_y - 2\,{}^{S}B_{x,k}A_{xz}V_z - 2\,{}^{S}B_{y,k}A_{xy}V_x - 2\,{}^{S}B_{y,k}A_{yy}V_y - 2\,{}^{S}B_{y,k}A_{yz}V_z$$

$$-2\,{}^{S}B_{z,k}A_{xz}V_x - 2\,{}^{S}B_{z,k}A_{yz}V_y - 2\,{}^{S}B_{z,k}A_{zz}V_z$$

$$+A_{xx}V_x{}^2 + A_{yy}V_y{}^2 + A_{zz}V_z{}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + 2A_{yz}V_yV_z - B^2 \quad \textbf{(119)}$$

Simplifying and returning to matrix format gives:

$$r_k = \begin{pmatrix} {}^{S}B_{x,k}{}^{2} \\ 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k} \\ 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k} \\ {}^{S}B_{y,k}{}^{2} \\ 2\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} \\ {}^{S}B_{z,k}{}^{2} \\ {}^{S}B_{x,k} \\ {}^{S}B_{y,k} \\ {}^{S}B_{z,k} \\ 1 \end{pmatrix}^{T} \begin{pmatrix} A_{xx} \\ A_{xy} \\ A_{xz} \\ A_{yy} \\ A_{yz} \\ A_{zz} \\ -2A_{xx}V_x - 2A_{xy}V_y - 2A_{xz}V_z \\ -2A_{xy}V_x - 2A_{yy}V_y - 2A_{yz}V_z \\ -2A_{xz}V_x - 2A_{yz}V_y - 2A_{zz}V_z \\ A_{xx}V_x{}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}V_y{}^2 + 2A_{yz}V_yV_z + A_{zz}V_z{}^2 - B^2 \end{pmatrix} \quad \textbf{(120)}$$

The right-hand side of equation (120) is defined to be the 10×1 solution vector $\boldsymbol{\beta}$:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \end{pmatrix} = \begin{pmatrix} A_{xx} \\ A_{xy} \\ A_{xz} \\ A_{yy} \\ A_{yz} \\ A_{zz} \\ -2A_{xx}V_x - 2A_{xy}V_y - 2A_{xz}V_z \\ -2A_{xy}V_x - 2A_{yy}V_y - 2A_{yz}V_z \\ -2A_{xz}V_x - 2A_{yz}V_y - 2A_{zz}V_z \\ A_{xx}V_x{}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}V_y{}^2 + 2A_{yz}V_yV_z + A_{zz}V_z{}^2 - B^2 \end{pmatrix} \quad \textbf{(121)}$$

$$= \begin{pmatrix} A_{xx} \\ A_{xy} \\ A_{xz} \\ A_{yy} \\ A_{yz} \\ A_{zz} \\ -2\beta_0V_x - 2\beta_1V_y - 2\beta_2V_z \\ -2\beta_1V_x - 2\beta_3V_y - 2\beta_4V_z \\ -2\beta_2V_x - 2\beta_4V_y - 2\beta_5V_z \\ A_{xx}V_x{}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}V_y{}^2 + 2A_{yz}V_yV_z + A_{zz}V_z{}^2 - B^2 \end{pmatrix} \quad \textbf{(122)}$$

Equation (120) for the error residual r_k whose squared sum is to be minimized is then:

$$r_k = \begin{pmatrix} {^S B_{x,k}}^2 \\ 2\ {^S B_{x,k}}\ {^S B_{y,k}} \\ 2\ {^S B_{x,k}}\ {^S B_{z,k}} \\ {^S B_{y,k}}^2 \\ 2\ {^S B_{y,k}}\ {^S B_{z,k}} \\ {^S B_{z,k}}^2 \\ {^S B_{x,k}} \\ {^S B_{y,k}} \\ {^S B_{z,k}} \\ 1 \end{pmatrix}^T \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \end{pmatrix} \quad \textbf{(123)}$$

The $M \times 1$ error residual vector $\boldsymbol{r}$ from M magnetometer measurements is defined as:

$$\boldsymbol{r} = \begin{pmatrix} r_0 \\ r_1 \\ \ldots \\ r_{M-1} \end{pmatrix} \quad \textbf{(124)}$$

and the $M \times 10$ measurement matrix $\boldsymbol{X}$ containing the M measurements is defined as:

$$\boldsymbol{X} = \begin{pmatrix} {^S B_{x,0}}^2 & 2\ {^S B_{x,0}}\ {^S B_{y,0}} & \ldots & {^S B_{y,0}} & {^S B_{z,0}} & 1 \\ {^S B_{x,1}}^2 & 2\ {^S B_{x,1}}\ {^S B_{y,1}} & \ldots & {^S B_{y,1}} & {^S B_{z,1}} & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 1 \\ {^S B_{x,M-1}}^2 & 2\ {^S B_{x,M-1}}\ {^S B_{y,M-1}} & \ldots & {^S B_{y,M-1}} & {^S B_{z,M-1}} & 1 \end{pmatrix} \quad \textbf{(125)}$$

Equation (123) can then be expanded to represent M measurements as:

$$\boldsymbol{r} = \boldsymbol{X\beta} \quad \textbf{(126)}$$

The model being fitted is the homogeneous model $\boldsymbol{X\beta} = 0$ which can be solved for $\boldsymbol{\beta}$ using the eigen-decomposition approach described in Section 2.

The 10×10 matrix $\boldsymbol{X}^T\boldsymbol{X}$ whose eigenvectors and eigenvalues are to be determined evaluates to:

$$\boldsymbol{X}^T\boldsymbol{X} = \sum_{k=0}^{M-1} \begin{pmatrix} {}^{S}B_{x,k}^{\;4} & 2\,{}^{S}B_{x,k}^{\;3}\,{}^{S}B_{y,k} & 2\,{}^{S}B_{x,k}^{\;3}\,{}^{S}B_{z,k} & \dots & {}^{S}B_{x,k}^{\;2}\,{}^{S}B_{z,k} & {}^{S}B_{x,k}^{\;2} \\ 2\,{}^{S}B_{x,k}^{\;3}\,{}^{S}B_{y,k} & 4\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k}^{\;2} & 4\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & \dots & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k} \\ 2\,{}^{S}B_{x,k}^{\;3}\,{}^{S}B_{z,k} & 4\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & 4\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{z,k}^{\;2} & \dots & 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k} \\ {}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}^{\;3} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}^{\;2}\,{}^{S}B_{z,k} & \dots & {}^{S}B_{y,k}^{\;2}\,{}^{S}B_{z,k} & {}^{S}B_{y,k}^{\;2} \\ 2\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & 4\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}^{\;2}\,{}^{S}B_{z,k} & 4\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k}^{\;2} & \dots & 2\,{}^{S}B_{y,k}\,{}^{S}B_{z,k}^{\;2} & 2\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} \\ {}^{S}B_{x,k}^{\;2}\,{}^{S}B_{z,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k}^{\;3} & \dots & {}^{S}B_{z,k}^{\;3} & {}^{S}B_{z,k}^{\;2} \\ {}^{S}B_{x,k}^{\;3} & 2\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k} & 2\,{}^{S}B_{x,k}^{\;2}\,{}^{S}B_{z,k} & \dots & {}^{S}B_{x,k}\,{}^{S}B_{z,k} & {}^{S}B_{x,k} \\ {}^{S}B_{x,k}^{\;2}\,{}^{S}B_{y,k} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & \dots & {}^{S}B_{y,k}\,{}^{S}B_{z,k} & {}^{S}B_{y,k} \\ {}^{S}B_{x,k}^{\;2}\,{}^{S}B_{z,k} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k}\,{}^{S}B_{z,k} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k}^{\;2} & \dots & {}^{S}B_{z,k}^{\;3} & {}^{S}B_{z,k} \\ {}^{S}B_{x,k}^{\;2} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{y,k} & 2\,{}^{S}B_{x,k}\,{}^{S}B_{z,k} & \dots & {}^{S}B_{z,k} & 1 \end{pmatrix} \tag{127}$$

Because the eigenvalues of $\boldsymbol{X}^T\boldsymbol{X}$ are equal to the fit errors associated with the ten-parameter eigenvector solutions, the required solution vector $\boldsymbol{\beta}_{min}$ is the eigenvector associated with the smallest eigenvalue λ_{min}.

6.2 Ellipsoid Fit Matrix

The ellipsoid fit matrix $\boldsymbol{A}$ is computed from the first six rows of the solution vector $\boldsymbol{\beta}$ in equation (121):

$$\boldsymbol{A} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix} \tag{128}$$

The solution eigenvector $\boldsymbol{\beta}$ is undefined within a multiplicative factor of ±1 (assuming it is normalized to unit magnitude). Since physically sensible solutions for $\boldsymbol{A}$ require that it have a positive determinant, the entire solution vector $\boldsymbol{\beta}$ should be negated if $|\boldsymbol{A}| < 0$.

For the same reasons as for the seven-parameter calibration algorithm, the determinant of $\boldsymbol{A}$ is set to 1.0 and the geomagnetic field strength is computed with this assumption.

6.3 Hard Iron Vector

Rows 6 to 8 of equation (121) can be written as:

$$\begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \end{pmatrix} = -2 \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = -2\boldsymbol{A} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} \tag{129}$$

The solution for the hard iron vector $\boldsymbol{V}$ is:

$$\boldsymbol{V} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = -\left(\frac{1}{2}\right)\begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix}^{-1}\begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \end{pmatrix} = -\left(\frac{1}{2}\right)\boldsymbol{A}^{-1}\begin{pmatrix} \beta_6 \\ \beta_7 \\ \beta_8 \end{pmatrix} \quad \textbf{(130)}$$

The solution for the hard iron vector $\boldsymbol{V}$ is independent of any sign change of the solution vector $\boldsymbol{\beta}$.

6.4 Inverse Soft Iron Matrix

The inverse soft iron matrix $\boldsymbol{W}^{-1}$ is computed from the square root of the symmetric matrix $\boldsymbol{A}$:

$$\boldsymbol{W}^{-1} = \sqrt{\boldsymbol{A}} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{xy} & A_{yy} & A_{yz} \\ A_{xz} & A_{yz} & A_{zz} \end{pmatrix}^{\frac{1}{2}} = \begin{pmatrix} \beta_0 & \beta_1 & \beta_2 \\ \beta_1 & \beta_3 & \beta_4 \\ \beta_2 & \beta_4 & \beta_5 \end{pmatrix}^{\frac{1}{2}} \quad \textbf{(131)}$$

The matrix square root is computed by eigen-decomposition of the 3×3 ellipsoid matrix $\boldsymbol{A}$. By definition, the 3×3 matrix $\boldsymbol{Q}$ containing the eigenvectors and the 3×3 diagonal matrix $\boldsymbol{\Lambda}$ containing the eigenvalues of $\boldsymbol{A}$ are related by:

$$\boldsymbol{AQ} = \boldsymbol{Q\Lambda} \Rightarrow \boldsymbol{A} = \boldsymbol{Q\Lambda Q}^{-1} \Rightarrow \boldsymbol{\Lambda} = \boldsymbol{Q}^{-1}\boldsymbol{AQ} \quad \textbf{(132)}$$

The matrix $\boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1}$ can be shown to be the required square root of $\boldsymbol{A}$ by simple multiplication and using the standard result that the eigenvectors of a symmetric matrix are orthonormal:

$$\left(\boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1}\right)\left(\boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1}\right) = \boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1}\boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1} = \boldsymbol{Q\Lambda Q}^{-1} = \boldsymbol{A} \quad \textbf{(133)}$$

The required square-root solution for the inverse soft iron matrix is then:

$$\boldsymbol{W}^{-1} = \sqrt{\boldsymbol{A}} = \boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{-1} = \boldsymbol{Q}\sqrt{\boldsymbol{\Lambda}}\boldsymbol{Q}^{T} \quad \textbf{(134)}$$

6.5 Geomagnetic Field Strength

The geomagnetic field strength can be computed from the last component of equation (121):

$$B = \sqrt{A_{xx}{V_x}^2 + 2A_{xy}V_xV_y + 2A_{xz}V_xV_z + A_{yy}{V_y}^2 + 2A_{yz}V_yV_z + A_{zz}{V_z}^2 - \beta_9} \quad \textbf{(135)}$$

6.6 Fit Error

In the same manner as for the seven-parameter calibration model, the normalized fit error is defined as:

$$\varepsilon = \frac{1}{2B^2}\sqrt{\frac{\lambda_{min}}{M}} \tag{136}$$

and the percentage fit error $\varepsilon\%$ defined as:

$$\varepsilon\% = \frac{50}{B^2}\sqrt{\frac{\lambda_{min}}{M}} \tag{137}$$

7. Iterative Time Slice Solution

Previous versions of the Sensor Fusion library ran using three tasks running on top of the MQX-Lite Real Time Operating System (RTOS). The highest priority task was the high frequency (200 Hz or higher) sensor read task. The second highest priority task was the sensor fusion Kalman filter (typically running at 25 Hz) and the lowest priority task was the background magnetic calibration (running occasionally every few minutes).

The requirement for an RTOS was eliminated in the current sensor fusion release which uses a single task (typically running at 25 Hz) to implement the sensor fusion Kalman filter. The FIFO buffers in the sensors (typically 32 measurements long) capture of sensor data at the maximum rate permitted by the sensors and implement the previous high priority sensor read task but without interrupting the Kalman filter task. The magnetic calibration algorithms described in this document are now incrementally computed in time slices called every pass of the Kalman filter. Other than the decomposition into time slices there are no changes to the magnetic calibration mathematics.

8. Legal information

8.1 Definitions

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9. Contents

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